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LETTER TO THE EDITOR

Frequency-dependent deformation of the Hall plateaux

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Abstract. We calculate the Hall conductivity in the low frequency limit for a 2D model of independent electrons in the tails of the density of states between the disorder broadened Landau levels. It is shown that there are corrections to the quantized values of σ_{yx} which are proportional to ω^2 and to the number of localized states in a given tail of the band. We relate our theoretical results to microwave experiments on GaAs–AlGaAs heterostructures.

In a recent article [1] we have calculated the frequency dependence of the Hall conductivity in the lowest localization regime of a 2D as well as a 3D non-interacting electron gas. The question arises as to whether the result in 2D may be generalized and applied to regimes in which the Hall conductivity is quantized at $\omega = 0$.

Experimental data from transmission measurements of GaAs–AlGaAs heterostructures in microwave guides [2–4] indicate that the Hall plateaux seem to persist up to frequencies above 30 GHz. Nevertheless, their width decreases with increasing frequency and within the experimental accuracy it is not possible to decide whether the plateaux are really flat or whether they develop a finite slope.

We consider non-interacting electrons in 2D under the influence of a perpendicular magnetic field B, i.e. a system described by the one-particle Hamiltonian

$$H = H_0 + V(r)$$
$$H_0 = (1/2m)(p - eA)^2 \qquad A = \tfrac{1}{2}B(-y, x, 0) \tag{1}$$

with a white noise random potential V

$$\overline{V(r)} = 0 \qquad \overline{V(r_1)V(r_2)} = \lambda\delta(r_1 - r_2). \tag{2}$$

The magnetic field is assumed to be strong in the sense that the disorder-induced broadening of the Landau levels (LLs) is small compared with the Landau level distance. Consequently we neglect interactions between different subbands and discuss only the lowest Landau level (LLL). In the tail region the density of states reads

$$\rho(E) = (2/\sqrt{\pi})(1/2\pi l^2)(1/\Gamma)(|\varepsilon|/\Gamma)^2 \exp[-(|\varepsilon|/\Gamma)^2] \tag{3}$$

with $\varepsilon = E - \hbar\omega_c/2$ and $\Gamma = (2\pi l^2/\lambda)^{-1/2}$ and l denotes the magnetic length. For later use let us recall the low frequency asymptotics of the Kubo conductivities obtained in [1] for the Fermi energy being situated in the lower tail of the LLL, i.e. $E_F \ll \hbar\omega_c/2$.

$$\sigma_{xx}(\omega) = -ie^2\omega 2l^2\rho(E_F) \qquad \sigma_{yx}(\omega) = (e^2/h)(\hbar^2\omega^2/\Gamma^2)8\pi l^2 n \tag{4}$$

where n denotes the number of electrons per unit area. These results have been obtained in the one-instanton approximation following the method developed by Affleck [5] for the calculation of the density of states.

One of the main difficulties in previous discussions of instantons between two LLs has been the occurrence of edge states. A confining potential has to be taken into account if the Kubo conductivities are calculated in coordinate representation which implies the use of the equation of motion $i\hbar v_\mu = [r_\mu, H]$. We want to present a symmetry argument which allows us to circumvent difficulties due to edge states and confining potentials on the one hand and to avoid the explicit calculation of contributions from extended states on the other hand. For that purpose we establish the relation between the complex conductivity $\sigma(\omega)$

$$\sigma(\omega) = \beta \int_0^\infty e^{i\omega t - \eta t} \langle j; j(t)\rangle \, dt \qquad j = (1/\sqrt{2})(j_x + ij_y)$$

$$\sigma_{xx}(\omega) = \tfrac{1}{2}(\sigma(\omega) + \sigma^*(-\omega)) \qquad \sigma_{yx}(\omega) = (i/2)(\sigma(\omega) - \sigma^*(-\omega)) \tag{5}$$

and the force–force correlation γ_F

$$\gamma_F(\omega) = \int_0^\infty e^{i\omega t - \eta t} \langle F; F(t)\rangle \, dt \qquad F = -\frac{1}{\sqrt{2}}\frac{e}{m}\left(\frac{\partial V}{\partial x} + i\frac{\partial V}{\partial y}\right) \tag{6}$$

which is given by the identity

$$\sigma(\omega) = (e^2 n/m)\, i(\omega - \omega_c) + \gamma_F(\omega)/(\omega - \omega_c)^2. \tag{7}$$

In equations (5–7) we have used standard textbook notation [6] and the correlations are defined in terms of the Kubo scalar product

$$\langle A; B\rangle = \beta^{-1}\mathrm{Tr} \int_0^\beta d\lambda\, \rho(H) A(-i\hbar\lambda) B^+ \qquad \beta^{-1} = k_B T. \tag{8}$$

In the following it will be shown that the frequency dependent force–force correlation $\gamma_F(\omega)$ for the fully occupied LLL vanishes. Assuming for a moment the validity of this statement we are able to deduce from equation (7) that in the right localized tail of the LLL, i.e. for $\Gamma \ll \varepsilon_F \ll \hbar\omega_c/2$, $\varepsilon_F = E_F - \hbar\omega_c/2$ the Hall conductivity is

$$\sigma_{yx}(\omega; \hbar\omega_c/2 + \varepsilon_F) = (e^2/h)\, 1/(1 - \omega^2/\omega_c^2) - \sigma_{yx}(\omega; \hbar\omega_c/2 - \varepsilon_F). \tag{9}$$

The first term in the RHS of equation (9) is the Hall conductivity of the completely filled LLL and the second term gives the correction from localized states in leading order of the frequency expansion. In deriving equation (9) we also used the fact that the Hall conductivity can be decomposed into a Fermi edge contribution $\sigma_{yx}^{(-)}$ and an energy integral $\sigma_{yx}^{(+)}$ (defined in [1])—the former being antisymmetric and the latter being symmetric with respect to the band centre in the energy region under consideration. In order to obtain the final result we just have to insert $\sigma_{yx}(\omega)$ from equation (4) with n replaced by $-n + \frac{1}{2}\pi l^2$ and for comparison with the experiments the non-resolved spin degeneracy has to be taken into account.

We now want to prove that $\gamma(\omega; E_F \to \infty) = 0$ and thus equation (9) indeed holds in the one-band model, we write the susceptibility χ_{FF} related to the force–force correlation by

$$\gamma_F(\omega) = (\chi_{FF}(\omega) - \chi_{FF}(0))/i\omega \tag{10}$$

in terms of Green's functions $G^\pm$

$$\chi_{FF}(\omega) = \frac{1}{2\pi i\, Ar} \int f(E)\, \mathrm{Tr}\{\overline{FG_\omega^+ F^+ (G^+ - G^-)} + \overline{F(G^+ - G^-) F^+ G_{-\omega}^-}\}\, dE \tag{11}$$

where the notation $G_\omega^\pm = G(E + \hbar\omega \pm i\eta)$ has been used, Ar is the area and f denotes the Fermi distribution. Shifting the energies by $\pm\hbar\omega/2$ it is possible to derive from equation (11)

$$\gamma_{\mathrm{F}}(\omega) = \gamma_{\mathrm{F}}^{(-)}(E_{\mathrm{F}}, \omega) - \gamma_{\mathrm{F}}^{(+)}(E_{\mathrm{F}}, \omega) \tag{12}$$

$$\gamma_{\mathrm{F}}^{(+)}(E_{\mathrm{F}}, \omega) = \frac{\hbar}{2\pi A r}\int f(E)\,\mathrm{Tr}\{\overline{FG^{+}_{\omega/2}F^{+}G^{+}_{-\omega/2}G^{+}} - \overline{FG^{+}_{\omega/2}G^{+}F^{+}G^{+}}$$
$$+ \overline{FG^{-}_{\omega/2}G^{-}F^{+}G^{-}_{-\omega/2}} - \overline{FG^{-}F^{+}G^{-}_{-\omega/2}G^{-}}\}\,\mathrm{d}E. \tag{13}$$

At $T = 0$ the function $\gamma_{\mathrm{F}}^{(-)}$, which has not been given explicitly above, just depends on states with energies in the interval $]-\hbar\omega/2 + E_{\mathrm{F}}, E_{\mathrm{F}} + \hbar\omega/2[$ around the Fermi energy and vanishes with the density of states as $E_{\mathrm{F}}/\Gamma \to \infty$ in the one-band approximation. $\gamma_{\mathrm{F}}^{(+)}$ vanishes in the case of complete filling because it satisfies the scaling relation

$$\gamma_{\mathrm{F}}^{(+)}(\omega) = \gamma_{\kappa\cdot\mathrm{F}}^{(+)}(\kappa \cdot \omega) \qquad \text{for} \qquad E_{\mathrm{F}} \to \infty. \tag{14}$$

This statement can be checked explicitly noting that in the functional integral representation in terms of supervectors [1] $\Phi = (s, \chi)$ we have

$$\begin{aligned} G^{\pm}(\varepsilon; r, r') &= \mp\mathrm{i}\int [\mathrm{d}\bar{\Phi}][\mathrm{d}\Phi]\chi(r)\bar{\chi}(r')\exp(-S) \\ S &= \mp\mathrm{i}\int \bar{\Phi}(\varepsilon - V \pm \mathrm{i}\eta)\Phi\,\mathrm{d}^2 r. \end{aligned} \tag{15}$$

The transformation $V \to \kappa V$, $\Phi \to \kappa^{-1/2}\Phi$ maps $G^{\pm}(\varepsilon)$ onto $\kappa G^{\pm}(\kappa\varepsilon)$. From this property we can deduce the scaling relation (14). Consequently, for complete filling $\gamma_{\mathrm{F}}^{(+)}$ is already independent of the impurity strength before averaging.

We arrive at the conclusion that the force–force correlation indeed vanishes for the completely filled LLL and conclude in agreement with equation (9) that in the energy region $\Gamma \ll \varepsilon_{\mathrm{F}} < \hbar\omega_{\rho}/2$ the Hall conductivity in the case of non-resolved spin degeneracy reads

$$\sigma_{yx}(\omega) = (e^2/h)[2/(1 - \omega^2/\omega_c^2) - (2\hbar\omega/\Gamma)^2(2 - \nu)] \tag{16}$$

where $\nu = 2\pi l^2 n$ is the filling factor and $1 - \nu/2$ is the fraction of unoccupied localized states in the upper tail.

Finally, we want to discuss the experimental implications of our result. In microwave experiments the measured bolometer signal from the transmitted radiation is proportional to $\sigma_{yx}^2(\omega)$. Let b denote the magnetic field measured from the centre of the plateau at zero frequency in Tesla, $\omega = \tilde{\omega} \times 10^3\,\mathrm{GHz}$, $n = \tilde{n}\,2 \times 10^{11}\,\mathrm{cm}^{-2}$. Using the relation $\Gamma^2 = (2/\pi)\hbar\omega_c(\hbar/\tau)$ between the bandwidth, the lifetime τ and the corresponding mobility at zero magnetic field $\mu = e\tau/m$, $m = 0.067 m_e$ for GaAs, $\mu = \tilde{\mu} \times 10^5\,\mathrm{cm}^2\,\mathrm{V}^{-1}\,\mathrm{s}^{-1}$, we obtain from equation (16) that in the vicinity of the $\nu = 2$ plateau

$$\sigma_{yx}^2(b; \tilde{\omega}) \simeq 4(e^4/h^2)[1 - (\tilde{\mu}\tilde{\omega}^2/\tilde{n}^2)b]. \tag{17}$$

According to equation (17) we think that the frequency effect near the centre of the plateaux should be described in terms of the finite slope the Hall conductivity begins to develop in the GHz range.

We are, of course, aware of the fact that our description is based on the assumption of short-range disorder although the disorder has long range in GaAs. We believe that this does not affect the qualitative applicability of our theory to experiment. However, in order to obtain exact quantitative predictions it is probably not sufficient to estimate the bandwidth from the $B = 0$ mobility which in general does not characterize a sample of a given material uniquely.

In [4] it was assumed that only the plateau width decreases when increasing the frequency and becomes zero at some critical value of ω. Properly speaking, the notion of a critical frequency is incompatible with our result that the plateaux are deformed for any finite frequency, and we think that the interpretation of the experiments of [4] given by the authors is not the only possible one. We also disagree with their conclusion that the frequency dependent deformation is the same for samples with different mobilities. It follows from equation (17) that the effect of low mobilities is to diminish the deformation of the Hall plateaux in the GHz range provided the overlap of adjacent LLs does not become too large. In high mobility samples plateaux will disappear at lower frequencies. Since in GaAs–AlGaAs heterostructures the magnetic field has to be lowered in order to reach higher LLs and the bandwidth Γ satisfies $\Gamma/\hbar\omega_c \propto B^{-1/2}$ it will be difficult to resolve the higher subbands in the low mobility samples.

We want to stress again that our result applies to the centre of the plateau region. However, in order to observe the predicted effect not only does the slope given in equation (17) have to be large enough, but so too does the width Δ_B of the region in which the linear b dependence still holds. Recently Kuchar *et al* [3] considered the broadening of the delocalization region near the centre of the LL by introducing a frequency equivalent length L_ν in analogy to the inelastic scattering length L_{in}, claiming that only for states with $L_{loc} < \min(L_{in}, L_\nu)$ does the localization remain effective. Since Kuchar *et al* take the centre of the LL as the starting point of their consideration while we describe the behaviour of the conductivities well within the localization regime, the two approaches can be considered complementary. Although we have demonstrated that between adjacent LLs the Hall conductivity can never exhibit exact plateaux at $\omega \neq 0$ the effect claimed in [4] clearly reduces Δ_B thereby affecting the measurement of the σ_{yx} slope.

In order to verify our theoretical prediction we propose to investigate the deformation of the Hall plateaux in a series of high precision measurements at different frequencies in the range of 30–150 GHz as well as for samples of different mobilities.

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